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Results on Canonical Cosine, Sine Transforms

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Abstract: **As generalization of the fractional Fourier transform (FRFT), the linear canonical transform (LCT) has been used in several areas, including optical analysis and signal processing. In this paper we have proved some important results about shifting property, differentiation property. Finally, the CTs of some selected functions are derived.**

Keywords: **Fourier transforms, LCT, Fractional Fourier Transform.**

1. INTRODUCTION

As a generalization of the classical Fourier transform and the fractional Fourier transform (FrFT), the linear canonical transform (LCT) receives much interest in recent years. Many important transforms, for example, the Fourier transform, the Fresnel transform, and the scaling operations are all special cases of the LCT. Recently further generalization of fractional Fourier transform known as linear canonical transform was introduced by Moshinsky [4] in 1971. Pei, Ding [5] had studied its eigen value aspect. In fact LCT is not only the generalization of the FRFT, but also the generalization of the many other integral transforms, like Fresnel transform, Chirp transform etc. Later on numbers of integral transforms are extended in its fractional domain. For examples Almeida [2] had studied fractional Fourier transform, Akay [1] developed fractional Mellin transform, Gudadhe A. S., Joshi A. V. [3], On Generalized Half Canonical Cosine Transform.

Linear canonical transform is a three parameter linear integral transform which has several special cases as fractional Fourier transform, Fresnel transform, Chirp transform etc. Linear canonical transform is defined as,

$$[LCTf(t)](s) = \sqrt{\frac{1}{2\pi ib}} \cdot \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \cdot e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot e^{-i\left(\frac{s}{b}\right)t} f(t)\,dt, \qquad \text{for } b \neq 0$$

$$= \sqrt{d}\; e^{\frac{i}{2}(cd)s^2} \cdot f(d \cdot s), \text{ for } b = 0, \text{ with } ad - bc = 1,$$

Where a, b, c, and d are real parameters independent on s and t.

2. DIRAC-DELTA FUNCTION

The delta function also called impulse function, well known to physicists as well as mathematicians, was introduced around the year 1920 by noble Laurate P.A.M. Dirac, while working on some quantum mechanical problems. Dirac, defined the delta function as, $\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty & t = 0 \end{cases}$

2.1 Canonical Transforms of Some Selected Generalized Functions: In this paper we have obtained the values of canonical cosine transform of some selected functions for which the results are used from the reference [1, 2]

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2.1.1 Canonical Cosine Transform of $\delta(t)$:

$$\{CCT\ [\delta(t)]\}(s)=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cdot\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot\cos\left(\frac{s}{b}t\right)\cdot\delta(t)dt$$

Using the definition of delta function given in 2

$$\{CCT\ [\delta(t)]\}(s)=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}$$

2.1.2 Canonical Cosine Transform of cos *t* :

$$\{CCT\ [(\cos t)]\}(s)=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot\cos\left(\frac{s}{b}t\right)\cdot\cos t\,dt.$$

$$\{CCT\ [(\cos t)]\}(s)=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \frac{1}{2}\left[\cos\left(\frac{s}{b}+1\right)t+\cos\left(\frac{s}{b}-1\right)t\right]dt$$

$$=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\left[\frac{1}{2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \cos\left(\frac{s}{b}+1\right)t\cdot dt+\frac{1}{2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \cos\left(\frac{s}{b}-1\right)t.dt\right]$$

$$=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cdot\left[\frac{1}{2}\left[\frac{\sqrt{\pi i}\cdot e^{\frac{-i\left(\frac{s}{b}+1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}}\right]+\frac{1}{2}\left[\frac{\sqrt{\pi i}\cdot e^{\frac{-i\left(\frac{s}{b}-1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}}\right]\right]$$

$$=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cdot\frac{1}{2}\left\{\frac{\sqrt{\pi i}\cdot e^{\frac{-ib\left(\frac{s}{b}+1\right)^2}{2a}}}{\sqrt{\frac{a}{2b}}}+\frac{\sqrt{\pi i}\cdot e^{\frac{-ib\left(\frac{s}{b}-1\right)^2}{2a}}}{\sqrt{\frac{a}{2b}}}\right\}$$

$$=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cdot\frac{1}{2}\frac{\sqrt{\pi i}\sqrt{2b}}{\sqrt{a}}\left[e^{\frac{-ib\left(\frac{s}{b}+1\right)^2}{2a}}+e^{\frac{-ib\left(\frac{s}{b}-1\right)^2}{2a}}\right]$$

$$\{CCT\ (\cos t\}(s)=\frac{e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}}{2\sqrt{a}}\left[e^{\frac{-ib\left(\frac{s}{b}+1\right)^2}{2a}}+e^{\frac{-ib\left(\frac{s}{b}-1\right)^2}{2a}}\right]$$

2.1.3 Canonical Cosine Transform of 1:

$$\{CCT\ [(1)]\}(s)=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\cdot\cos\left(\frac{s}{b}t\right)\cdot 1\cdot dt$$

$$=\sqrt{\frac{1}{2\pi ib}}\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}\ \cos\left(\frac{s}{b}t\right)\ dt$$

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$$= \sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \left[\sqrt{\pi i} \cdot \frac{e^{\frac{-i\left(\frac{s}{b}\right)^2}{\frac{2a}{b}}}}{\frac{\sqrt{a}}{\sqrt{2b}}} \right]$$

$$= \sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \left[\frac{\sqrt{2b} \cdot \sqrt{\pi i} \cdot e^{\frac{-\left(\frac{s^2}{b}\right)}{2a}}}{\sqrt{a}} \right]$$

$$= \frac{1}{\sqrt{a}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \cdot e^{-\frac{i}{2}\left[\frac{1}{ab}\right]s^2}$$

$$\{CCT\ [1]\}(s) = \frac{1}{\sqrt{a}} e^{\frac{i}{2}s^2\left[\frac{d}{b} - \frac{1}{ab}\right]}$$

3. CANONICAL SINE TRANSFORM OF SOME SELECTED GENERALIZED FUNCTIONS

3.3.1 Canonical Sine Transform of $\delta(t)$:

$$\{CST\ [\delta(t)]\}(s) = -i\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot \sin\left(\frac{s}{b}t\right) \delta(t) \cdot dt$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \cdot \left[e^0 \cdot \sin(0)\right] \qquad = (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} (0) = 0$$

Similarly we can calculated

$$\{CST\,\delta(t-\alpha)\}(s) = -\sqrt{\frac{1}{2\pi ib}} e^{\frac{i}{2}\frac{d}{b}s^2} e^{\frac{i}{2}\frac{a}{b}\alpha^2} \sin\frac{s}{b}\alpha$$

3.3.2 Canonical Sine Transform of sin *t*:

$$\{CST\ (\sin t)\}(s) = -\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot i \sin\left(\frac{s}{b}t\right) \sin t \cdot dt$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot \frac{1}{2}\left[\cos\left(\frac{s}{b}+1\right)t - \cos\left(\frac{s}{b}-1\right)t\right] dt$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \left[\frac{1}{2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cos\left(\frac{s}{b}+1\right)t\ dt - \frac{1}{2}\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cos\left(\frac{s}{b}-1\right)t\right] dt$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \frac{1}{2} \left[\frac{\sqrt{\pi i} \cdot e^{\frac{-i\left(\frac{s}{b}+1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}} - \frac{\sqrt{\pi i} \cdot e^{\frac{-i\left(\frac{s}{b}-1\right)^2}{4\frac{a}{2b}}}}{\sqrt{\frac{a}{2b}}} \right]$$

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$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \frac{1}{2}\left[\frac{\sqrt{2b}\sqrt{\pi i} \cdot e^{\frac{-i\left(\frac{s}{b}+1\right)^2}{\frac{2a}{b}}}}{\sqrt{a}} - \frac{\sqrt{\pi i}\sqrt{2b} \cdot e^{\frac{-i\left(\frac{s}{b}-1\right)^2}{4\frac{a}{2b}}}}{\sqrt{a}}\right]$$

$$= (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \frac{1}{2}\frac{\sqrt{2b}\sqrt{\pi i}}{\sqrt{a}}\left[e^{\frac{-i\left(\frac{s}{b}+1\right)^2 b}{2a}} - e^{\frac{-i\left(\frac{s}{b}-1\right)^2 b}{2a}}\right]$$

$$\{\ CST(\sin t)\}(s) = \frac{(-i)\cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}}{2\sqrt{a}}\left[e^{-i\left(\frac{s}{b}+1\right)^2 b} - e^{-i\left(\frac{s}{b}-1\right)^2 b}\right]$$

3.3.3 Canonical Sine Transform of 1:

$$\{CST(1)\}(s) = (-i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \cdot \sin\left(\frac{s}{b}t\right) dt$$

$$= (i)\sqrt{\frac{1}{2\pi ib}} \cdot e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \cdot (0)$$

$$\{CST(1)\}(s) = 0$$

4. CONCLUSION

In this paper, brief introduction of the generalized linear canonical transform is given and its Shifting property, differentiation property of linear canonical transform, LCT, CCT, CST of some selected functions obtained which will be useful in solving differential equations occurring in signal processing and many other branches of engineering.

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